CSC D70: Compiler Optimization Register Allocation

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The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons

Register Allocation and Coalescing

- Introduction
- Abstraction and the Problem
- Algorithm
- Spilling
- Coalescing

Reading: ALSU 8.8.4

Motivation

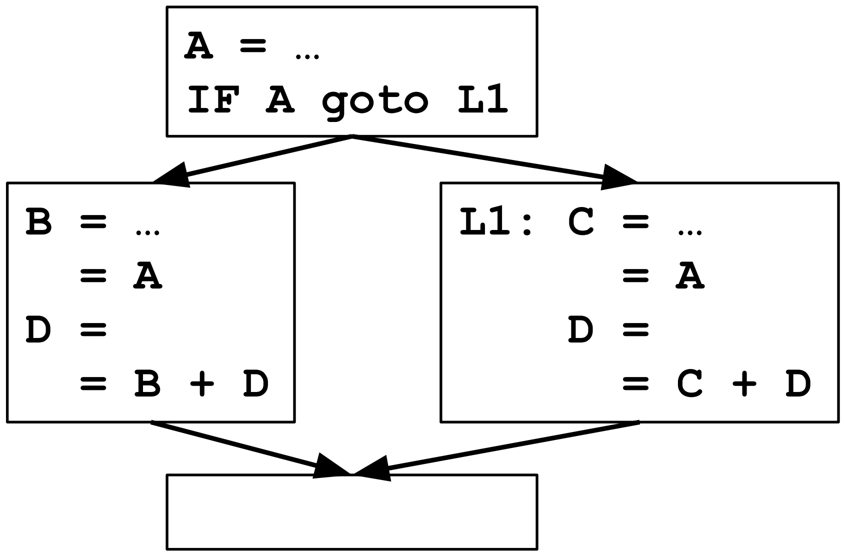
- **Problem**
  - Allocation of variables (pseudo-registers) to hardware registers in a procedure
- **A very important optimization!**
  - Directly reduces running time
    - (memory access → register access)
  - Useful for other optimizations
    - e.g. CSE assumes old values are kept in registers.

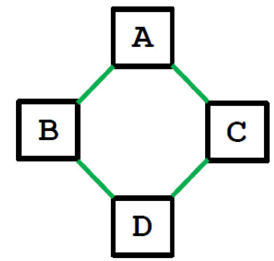
Goals

- Find an allocation for all pseudo-registers, if possible.

- If there are not enough registers in the machine, choose registers to spill to memory

Register Assignment Example





- Find an assignment (no spilling) with only 2 registers
  - A and D in one register, B and C in another one
- What assumptions?
  - After assignment, no use of A & (and only one of B and C used)

An Abstraction for Allocation & Assignment

- **Intuitively**
  - Two pseudo-registers **interfere** if at some point in the program they cannot both occupy the same register.

- **Interference graph**: an undirected graph, where
  - nodes = pseudo-registers
  - there is an edge between two nodes if their corresponding pseudo-registers interfere

- **What is not represented**
  - Extent of the interference between uses of different variables
  - Where in the program is the interference

Interfere many times vs. once

E.g., cold path vs. hot path

Register Allocation and Coloring

- A graph is **n-colorable** if:
  - every node in the graph can be colored with one of the n colors such that two adjacent nodes do not have the same color.

- Assigning n register (without spilling) = Coloring with n colors
  - assign a node to a register (color) such that no two adjacent nodes are assigned same registers (colors)

- Is spilling necessary? = Is the graph n-colorable?

- To determine if a graph is n-colorable is NP-complete, for n>2
  - Too expensive
  - Heuristics

Algorithm

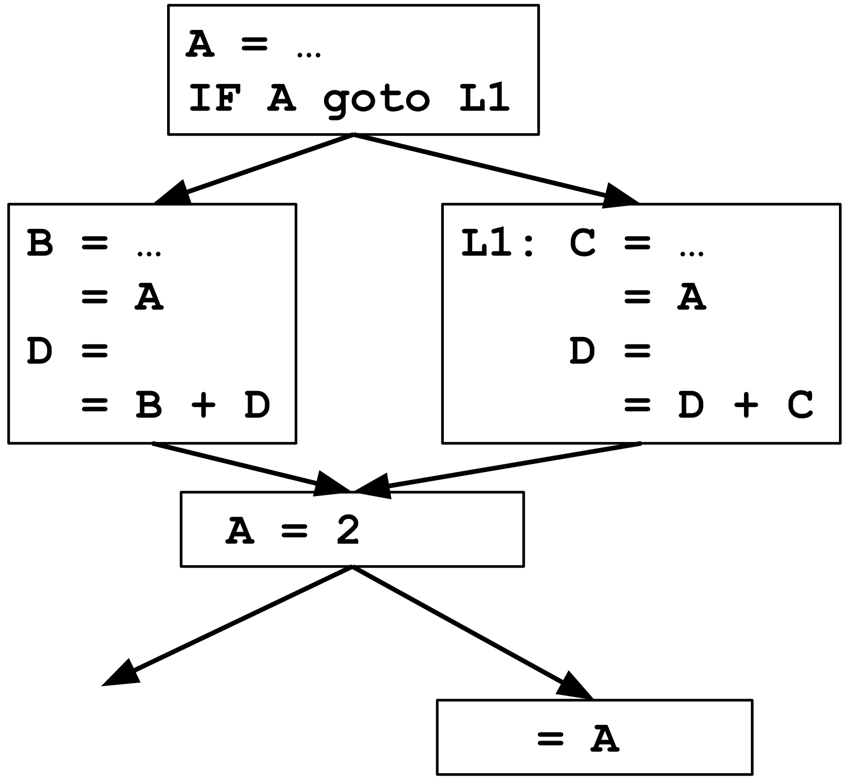
Step 1. Build an interference graph

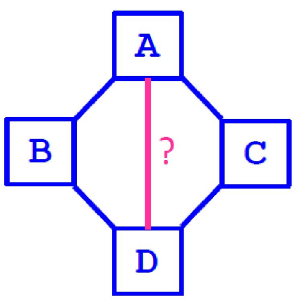
a. refining notion of a node
b. finding the edges

Step 2. Coloring

- use heuristics to try to find an n-coloring
  - Success:
    - colorable and we have an assignment
  - Failure:
    - graph not colorable, or
    - graph is colorable, but it is too expensive to color

Step 1a. Nodes in an Interference Graph





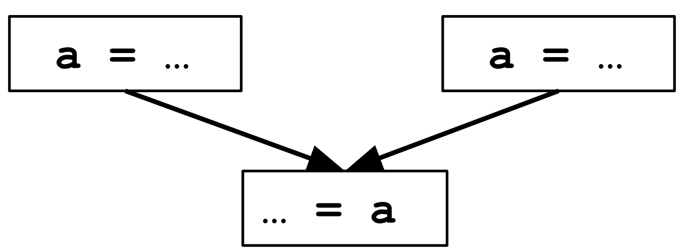
Interference Graph

Should we add A-D edge?

No, since new def of A

Live Ranges and Merged Live Ranges

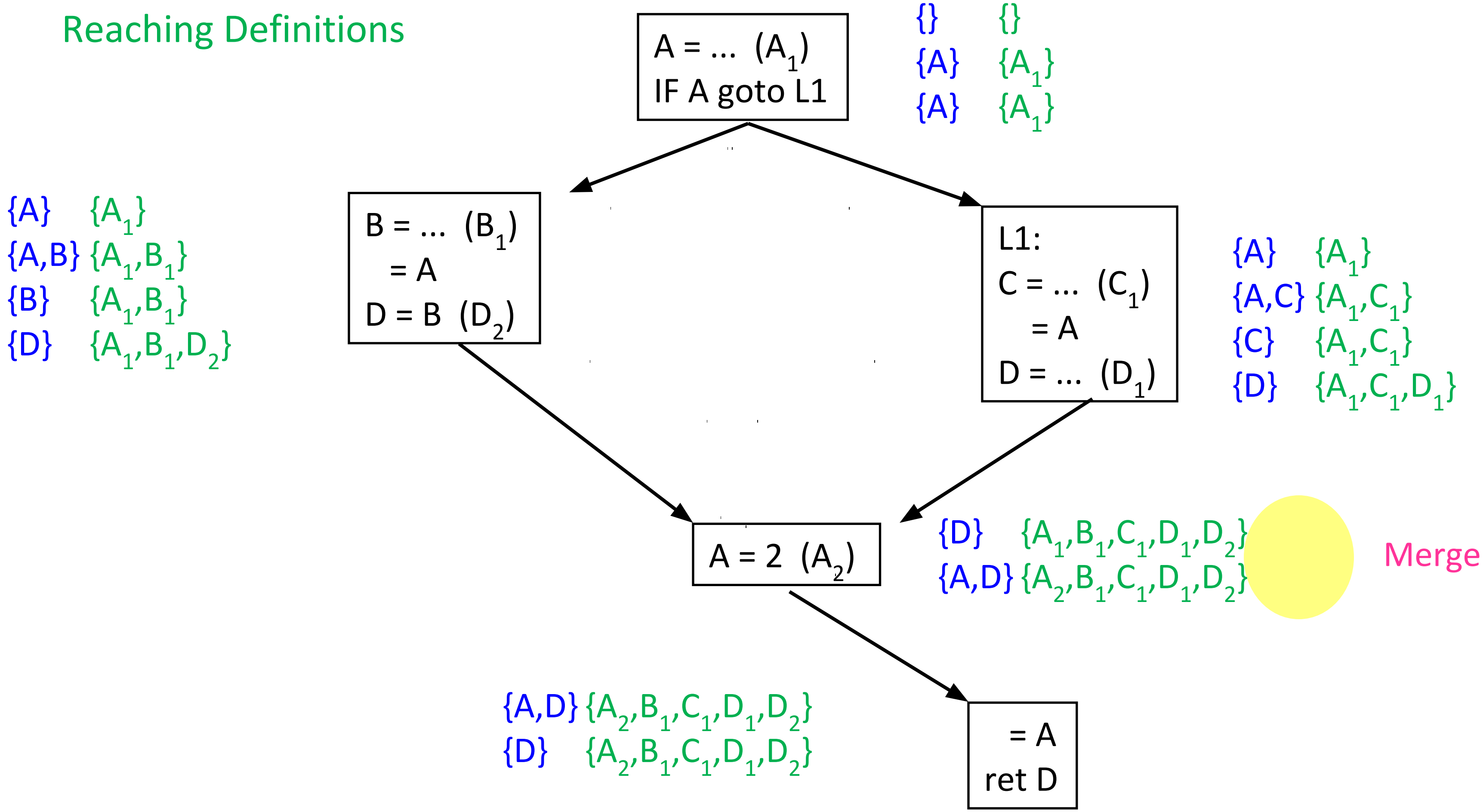
- **Motivation: to create an interference graph that is easier to color**
  - Eliminate interference in a variable's "dead" zones.
  - Increase flexibility in allocation:
    - can allocate same variable to different registers
- A **live range** consists of a definition and all the points in a program in which that definition is live.
  - How to compute a live range?
- Two overlapping live ranges for the **same** variable must be merged



Example (Revisited)

Live Variables

Reaching Definitions



Merging Live Ranges

- **Merging definitions into equivalence classes**
  - Start by putting each definition in a different equivalence class
  - Then, for each point in a program:
    - if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
      - merge the equivalence classes of all such definitions into one equivalence class
    - *(Sound familiar?)*

- **From now on, refer to merged live ranges simply as live ranges**
  - merged live ranges are also known as "webs"

SSA Revisited: What Happens to Φ Functions

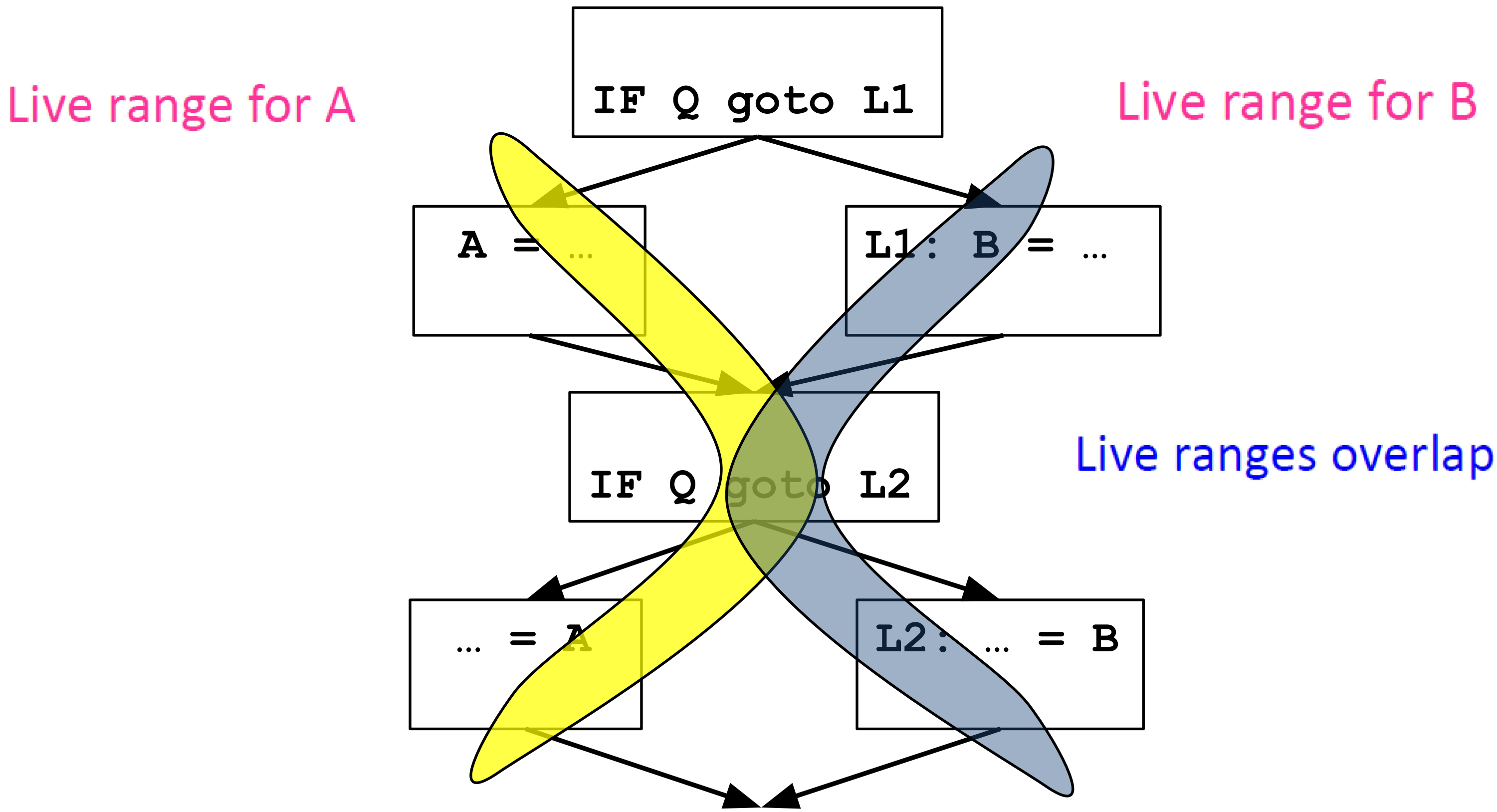
- Now we see why it is unnecessary to "implement" a Φ function
  - Φ functions and SSA variable renaming simply turn into merged live ranges
- When you encounter: $X_4 = \Phi(X_1, X_2, X_3)$
  - merge X_1, X_2, X_3, and X_4 into the same live range
  - delete the Φ function
- Now you have effectively converted back out of SSA form

Step 1b. Edges of Interference Graph

- **Intuitively**:
  - Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.
  - Algorithm:
    - At each point in the program:
      - enter an edge for every pair of live ranges at that point.

- **An optimized definition & algorithm for edges:**
  - Algorithm:
    - check for interference only at the start of each live range
  - Faster
  - Better quality

Live Range Example 2

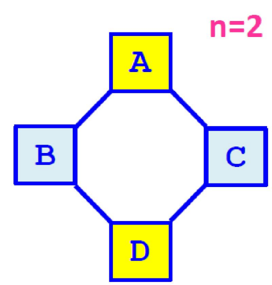


Because ranges overlap: Won't assign A and B to same register
(even though would have been ok: path sensitive vs. path insensitive analysis)

Step 2. Coloring

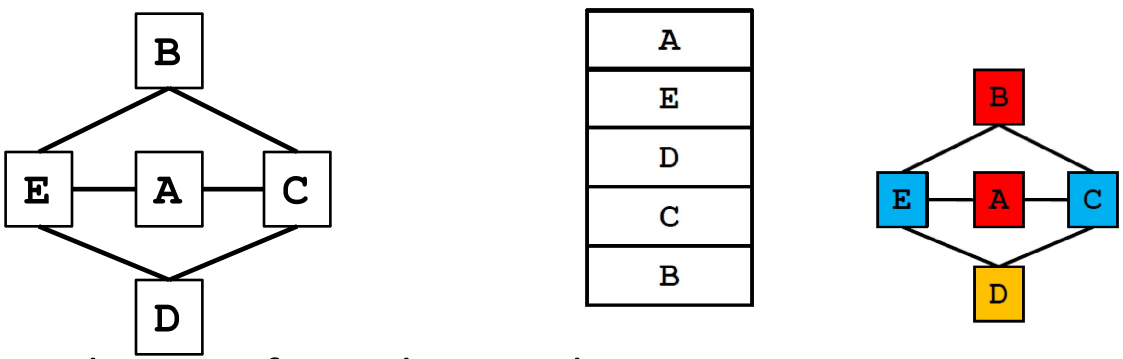
- **Reminder: coloring for n > 2 is NP-complete**

- **<u>Observations</u>:**
  - a node with degree < n ⇒
    - can always color it successfully, given its neighbors' colors
  - a node with degree = n ⇒
    - can only color if at least two neighbors share same color
  - a node with degree > n ⇒
    - maybe, not always



Coloring Algorithm

- Algorithm:
  - Iterate until stuck or done
    - Pick any node with degree < n
    - Remove the node and its edges from the graph
  - If done (no nodes left)
    - reverse process and add colors
- Example ($n = 3$):



- Note: degree of a node may drop in iteration
- Avoids making arbitrary decisions that make coloring fail

More details

- **Apply coloring heuristic**

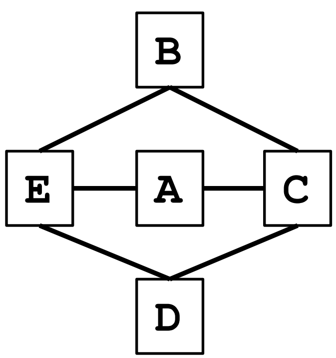
  Build interference graph
  Iterate until there are no nodes left
      If there exists a node v with less than n neighbor
          push v on register allocation stack
      else
          return (coloring heuristics fail)
      remove v and its edges from graph

- **Assign registers**

  While stack is not empty
      Pop v from stack
      Reinsert v and its edges into the graph
      Assign v a color that differs from all its neighbors

What Does Coloring Accomplish?

- **Done**:
  - colorable, also obtained an assignment
- **Stuck**:
  - colorable or not?



Extending Coloring: Design Principles

- **A pseudo-register is**
  - Colored successfully: allocated a hardware register
  - Not colored: left in memory
- **Objective function**
  - Cost of an uncolored node:
    - proportional to number of uses/definitions (dynamically)
    - estimate by its loop nesting
  - Objective: minimize sum of cost of uncolored nodes
- **Heuristics**
  - Benefit of spilling a pseudo-register:
    - increases colorability of pseudo-registers it interferes with
    - can approximate by its degree in interference graph
  - Greedy heuristic
    - spill the pseudo-register with lowest cost-to-benefit ratio, whenever spilling is necessary

Spilling to Memory

- CISC architectures
  - can operate on data in memory directly
  - memory operations are slower than register operations
- RISC architectures
  - machine instructions can only apply to registers
  - Use
    - must first load data from memory to a register before use
  - Definition
    - must first compute RHS in a register
    - store to memory afterwards
  - Even if spilled to memory, needs a register at time of use/definition

Chaitin: Coloring and Spilling

- **Identify spilling**

  Build interference graph
  Iterate until there are no nodes left
      If there exists a node v with less than n neighbor
          place v on stack to register allocate
      else
          v = node with highest degree-to-cost ratio
          mark v as spilled
      remove v and its edges from graph

- **Spilling may require use of registers; change interference graph**

  While there is spilling
  rebuild interference graph and perform step above

- **Assign registers**

  While stack is not empty
      Remove v from stack
      Reinsert v and its edges into the graph
      Assign v a color that differs from all its neighbors

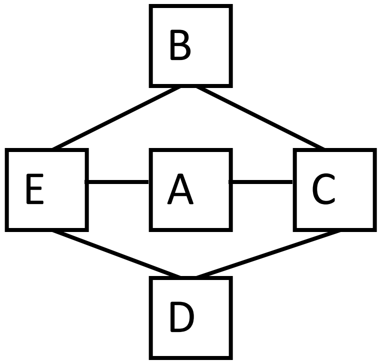
Spilling

- What should we spill?
  - Something that will eliminate a lot of interference edges
  - Something that is used infrequently
  - Maybe something that is live across a lot of calls?

- One Heuristic:
  - spill cheapest live range (aka "web")
  - Cost = $[(\#\ \text{defs}\ \&\ \text{uses}) * 10^{\text{loop-nest-depth}}]/\text{degree}$

Quality of Chaitin's Algorithm

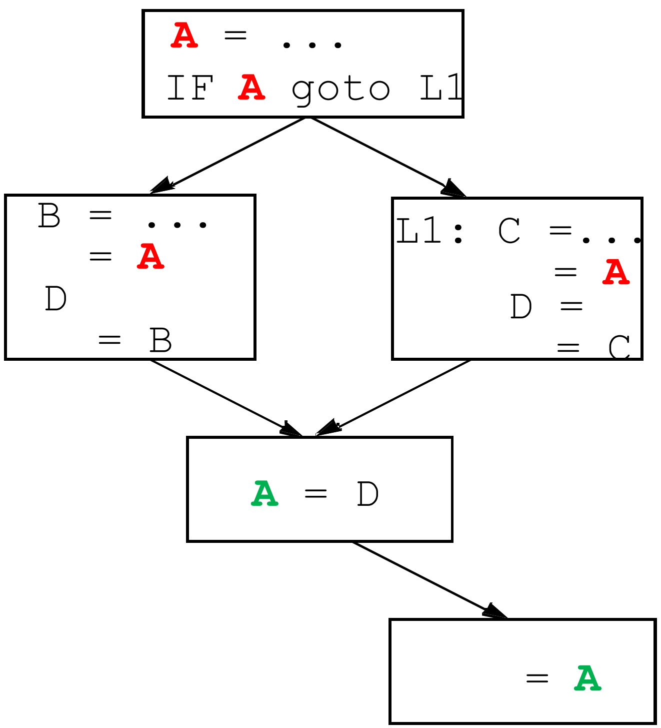
- Giving up too quickly
- N=2

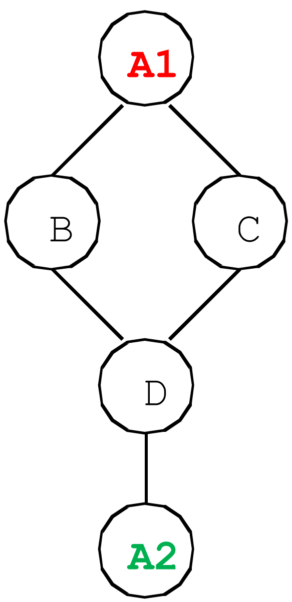


- An optimization: "Prioritize the coloring"
  - Still eliminate a node and its edges from graph
  - Do not commit to "spilling" just yet
  - Try to color again in assignment phase.

Splitting Live Ranges

- Recall: Split pseudo-registers into live ranges to create an interference graph that is easier to color
  - Eliminate interference in a variable's "dead" zones.
  - Increase flexibility in allocation:
    - can allocate same variable to different registers





Insight

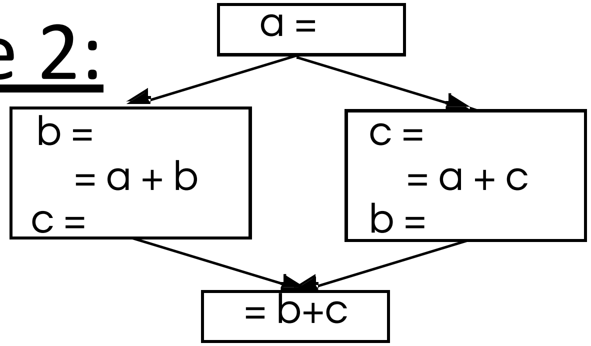
- Split a live range into smaller regions (by paying a small cost) to create an interference graph that is easier to color
  - Eliminate interference in a variable's "nearly dead" zones.
    - Cost: Memory loads and stores
      - Load and store at boundaries of regions with no activity
    - # active live ranges at a program point can be > # registers
  - Can allocate same variable to different registers
    - Cost: Register operations
      - a register copy between regions of different assignments
    - # active live ranges cannot be > # registers

Examples

Example 1:

```
FOR i = 0 TO 10
FOR j = 0 TO 10000
    A = A + ...
 (does not use B)
FOR j = 0 TO 10000
    B = B + ...
 (does not use A)
```

Example 2:



Example 1

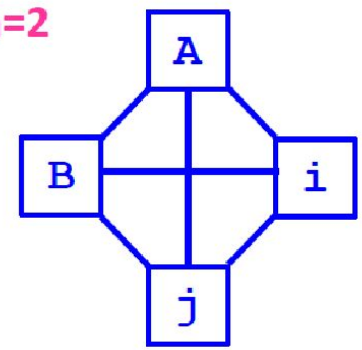
spill i

spill B

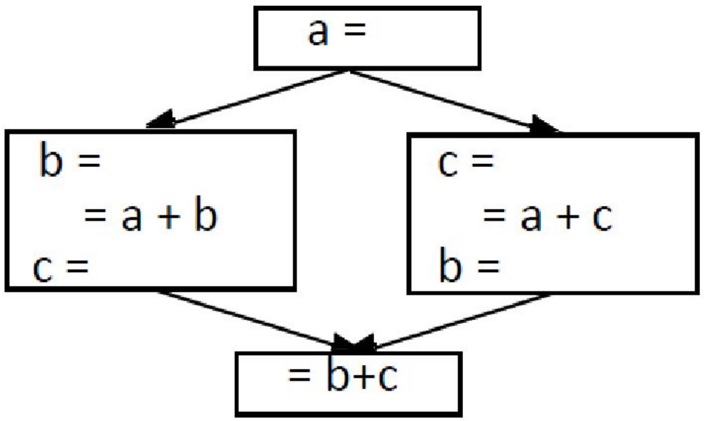
spill A

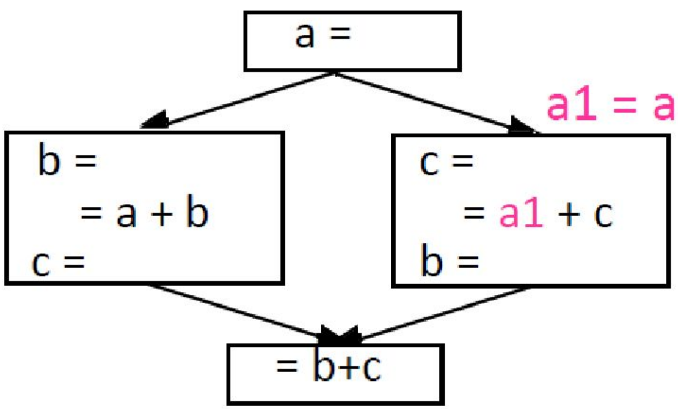
```
FOR i = 0 TO 10
    FOR j = 0 TO 10000
        A = A + ...
        (does not use B)
    FOR j = 0 TO 10000
        B = B + ...
        (does not use A)
```

n=2

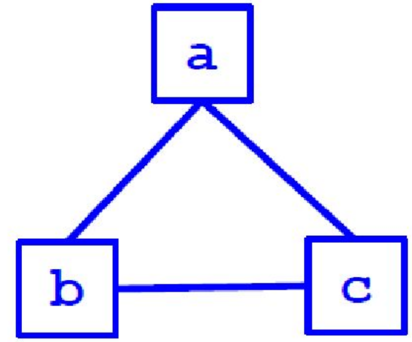


Example 2

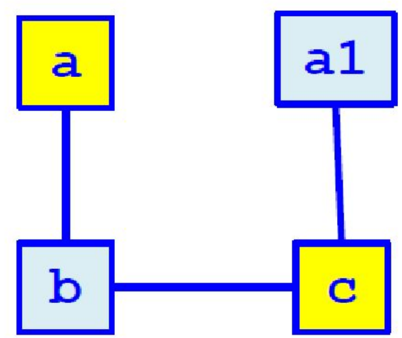




n=2



Can't 2-color



Can 2-color
("a" gets 2 regs)

Live Range Splitting

- When do we apply live range splitting?
- Which live range to split?
- Where should the live range be split?
- How to apply live-range splitting with coloring?
  - Advantage of coloring:
    - defers arbitrary assignment decisions until later
  - When coloring fails to proceed, may not need to split live range
    - degree of a node >= n does not mean that the graph definitely is not colorable
  - Interference graph does not capture positions of a live range

One Algorithm

- Observation: spilling is absolutely necessary if
  - number of live ranges active at a program point > n
- Apply live-range splitting before coloring
  - Identify a point where number of live ranges > n
  - For each live range active around that point:
    - find the outermost "block construct" that does not access the variable
  - Choose a live range with the largest inactive region
  - Split the inactive region from the live range

Summary

- **Problems:**
  - Given n registers in a machine, is spilling avoided?
  - Find an assignment for all pseudo-registers, whenever possible.
- **Solution:**
  - Abstraction: an **interference graph**
    - nodes: live ranges
    - edges: presence of live range at time of definition
  - Register Allocation and Assignment problems
    - equivalent to **n-colorability** of interference graph
      ➔ NP-complete
  - Heuristics to find an assignment for n colors
    - successful: colorable, and finds assignment
    - not successful: colorability unknown & no assignment

CSC D70: Compiler Optimization Register Coalescing

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Let’s Focus on Copy Instructions

```
X = A + B;
…
Y = X;
…
Z = Y + 4;
```

→

```
X = A + B;
…
// deleted
…
Z = X + 4;
```

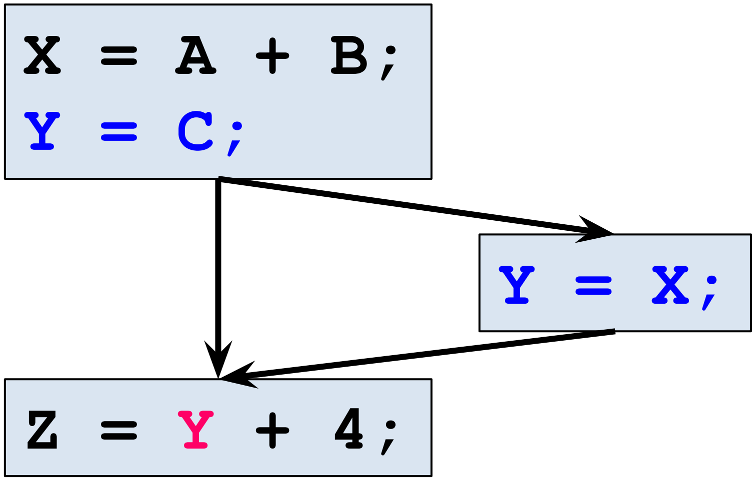
2. Dead Code Elimination

1. Copy Propagation

- Optimizations that help optimize away copy instructions:
  - Copy Propagation
  - Dead Code Elimination

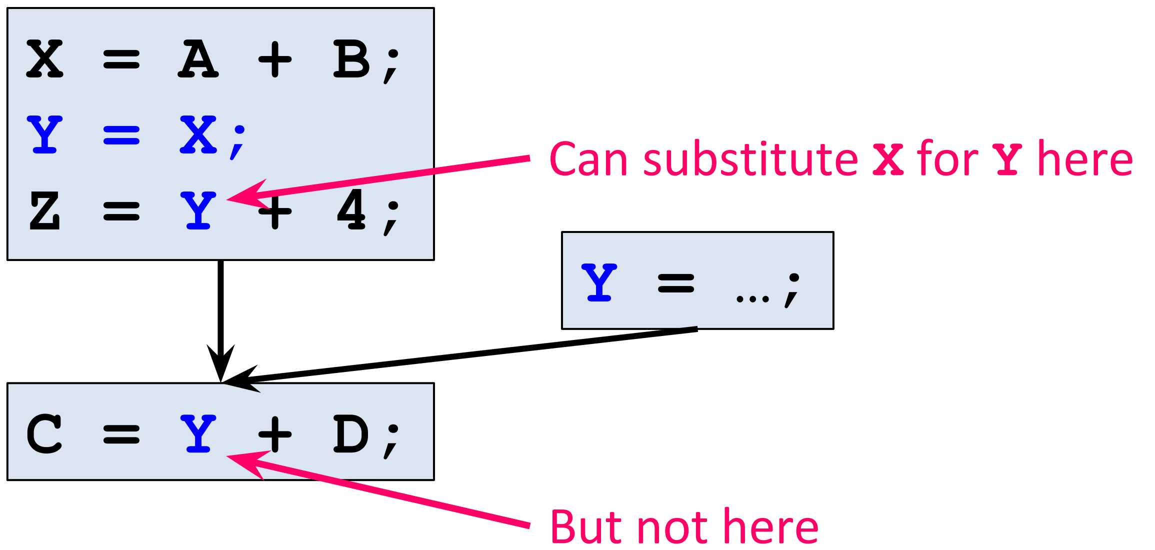
- Can all copy instructions be eliminated using this pair of optimizations?

Example Where Copy Propagation Fails



- Use of copy target has multiple (conflicting) reaching definitions

Another Example Where the Copy Instruction Remains



- Copy target (**Y**) still live even after some successful copy propagations

- Bottom line:
  - copy instructions may still exist when we perform register allocation

Copy Instructions and Register Allocation

- What clever thing might the register allocator do for copy instructions?

```
…
Y = X;
…
```

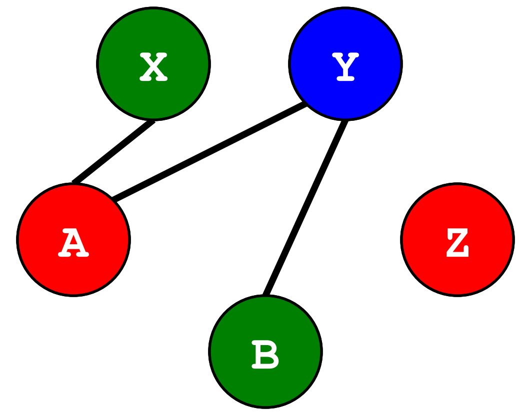
…
~~r7 = r7;~~
…

- If we can assign both the source and target of the copy to the same register:
  - then we don't need to perform the copy instruction at all!
  - the copy instruction can be removed from the code
    - even though the optimizer was unable to do this earlier

- <u>One way to do this:</u>
  - treat the copy source and target as the same node in the interference graph
    - then the coloring algorithm will naturally assign them to the same register
  - this is called "coalescing"

Simple Example: Without Coalescing

```
X = …;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```

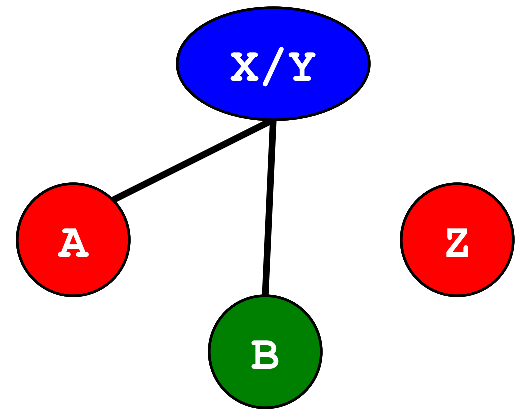


Valid coloring with 3 registers

- Without coalescing, **X** and **Y** can end up in different registers
  - – cannot eliminate the copy instruction

Example Revisited: With Coalescing

```
X = …;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```

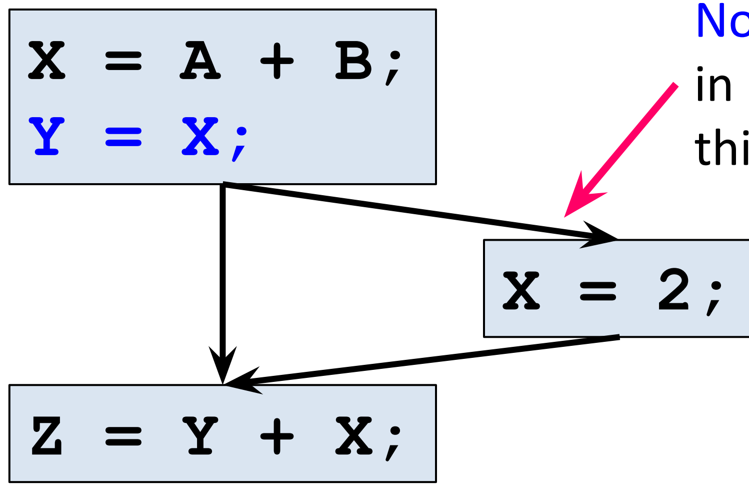


Valid coloring with 3 registers

- With coalescing, **X** and **Y** are now guaranteed to end up in the same register
  - the copy instruction can now be eliminated

- Great! So should we go ahead and do this for every copy instruction?

Should We Coalesce `X` and `Y` In This Case?



No! That would result in incorrect behavior if this branch is taken.

- It is legal to coalesce **`X`** and **`Y`** for a "**`Y = X`**" copy instruction iff:
  - initial definition of **`Y`**'s live range is this copy instruction, AND
  - the live ranges of **`X`** and **`Y`** do not interfere otherwise

- But just because it is legal doesn't mean that it is a good idea…

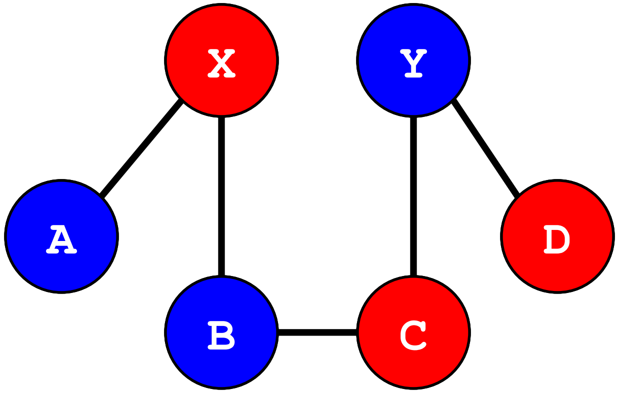
Why Coalescing May Be Undesirable

```
X = A + B;
…  // 100 instructions
Y = X;
…  // 100 instructions
Z = Y + 4;
```

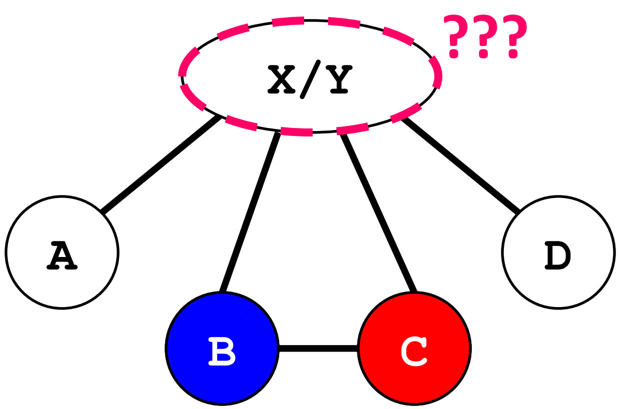
- What is the likely impact of coalescing **`X`** and **`Y`** on:
  - live range size(s)?
    - recall our discussion of live range splitting
  - colorability of the interference graph?
- Fundamentally, coalescing adds further constraints to the coloring problem
  - doesn't make coloring easier; may make it more difficult
- If we coalesce in this case, we may:
  - save a copy instruction, BUT
  - cause significant spilling overhead if we can no longer color the graph

When to Coalesce

- Goal when coalescing is legal:
  - coalesce *unless* it would make a colorable graph non-colorable
- The bad news:
  - predicting colorability is tricky!
    - it depends on the shape of the graph
    - graph coloring is NP-hard
- <u>Example</u>: assuming 2 registers, should we coalesce **`X`** and **`Y`**?



2-colorable

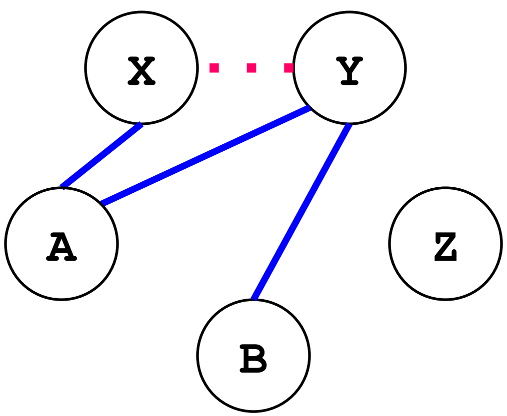


Not 2-colorable

Representing Coalescing Candidates in the Interference Graph

- To decide whether to coalesce, we augment the interference graph
- Coalescing candidates are represented by a new type of interference graph edge:
  - dotted lines: coalescing candidates
    - *try* to assign vertices the same color
      - (unless that is problematic, in which case they can be given different colors)
  - solid lines: interference
    - vertices *must* be assigned different colors

```
X = …;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```

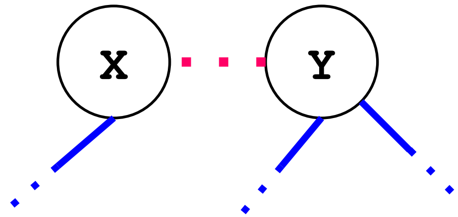


How Do We Know When Coalescing Will Not Cause Spilling?

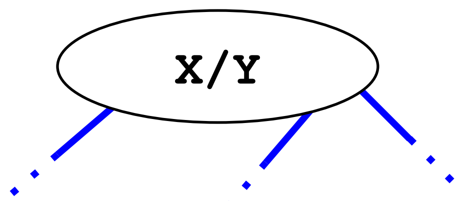
- Key insight:
  - Recall from the coloring algorithm:
    - we can always successfully N-color a node if its degree is < N
- To ensure that coalescing does not cause spilling:
  - check that the degree < N invariant is still locally preserved after coalescing
    - if so, then coalescing won't cause the graph to become non-colorable
  - no need to inspect the entire interference graph, or do trial-and-error
- Note:
  - We do NOT need to determine whether the full graph is colorable or not
  - Just need to check that coalescing does not cause a colorable graph to become non-colorable

Simple and Safe Coalescing Algorithm

- We can safely coalesce nodes **X** and **Y** if (|**X**| + |**Y**|) < N
  - Note: |**X**| = degree of node **X** counting interference (not coalescing) edges
- <u>Example</u>:



(|**X**| + |**Y**|) = (1 + 2) = 3



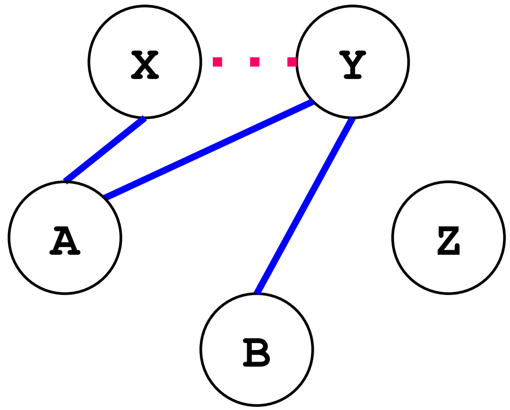
Degree of coalesced node can be no larger than 3

  - if N >= 4, it would always be safe to coalesce these two nodes
    - this cannot cause new spilling that would not have occurred with the original graph
  - if N < 4, it is unclear

How can we (safely) be more aggressive than this?

What About This Example?

- Assume N = 3
- Is it safe to coalesce **`X`** and **`Y`**?



(|**`X`**| + |**`Y`**|) = (1 + 2) = 3

(Not less than N)

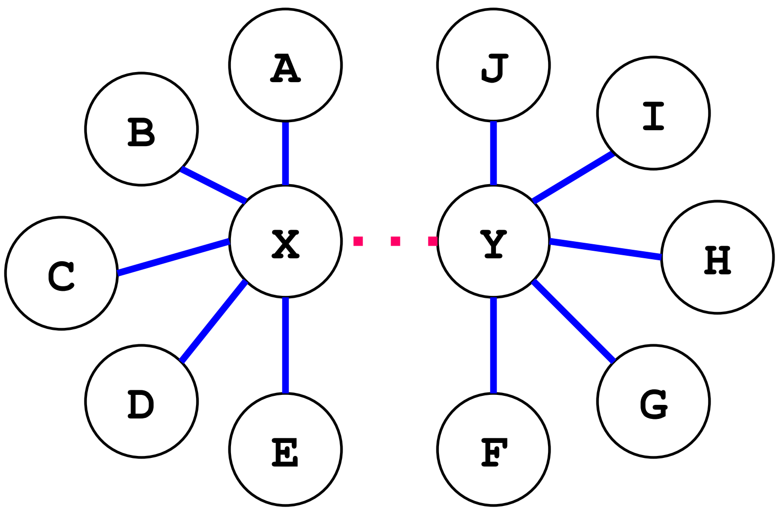
- <u>Notice</u>: **`X`** and **`Y`** share a common (interference) neighbor: node **`A`**
  - hence the degree of the coalesced **`X/Y`** node is actually 2 (not 3)
  - therefore coalescing **`X`** and **`Y`** *is* guaranteed to be safe when N = 3
- How can we adjust the algorithm to capture this?

Another Helpful Insight

- Colors are not assigned until nodes are popped off the stack
  - nodes with degree < N are pushed on the stack first
  - when a node is popped off the stack, we know that it can be colored
    - because the number of potentially conflicting neighbors must be < N
- Spilling only occurs if there is no node with degree < N to push on the stack

- Example: (N=2)

Another Helpful Insight



|**X**| = 5
|**Y**| = 5

2-colorable after coalescing **X** and **Y**?

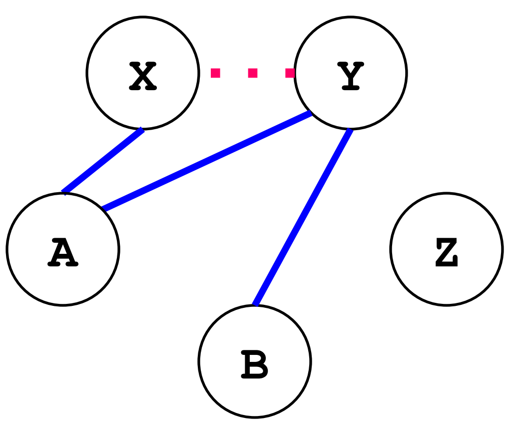
Building on This Insight

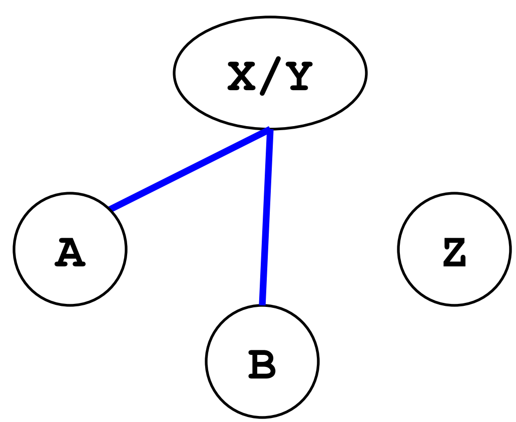
- When would coalescing cause the stack pushing (aka “simplification”) to get stuck?
  1. coalesced node must have a degree >= N
     - otherwise, it can be pushed on the stack, and we are not stuck
  2. AND it must have at least N neighbors that each have a degree >= N
     - otherwise, all neighbors with degree < N can be pushed before this node
       - reducing this node’s degree below N (and therefore we aren’t stuck)

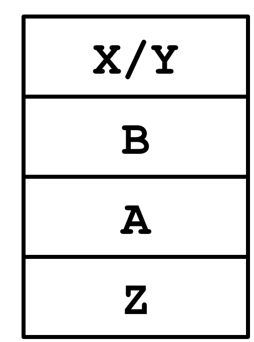
- To coalesce more aggressively (and safely), let’s exploit this second requirement
  - which involves looking at the degree of a coalescing candidate’s neighbors
    - not just the degree of the coalescing candidates themselves

Briggs's Algorithm

- Nodes **X** and **Y** can be coalesced if:
  – (number of neighbors of **X/Y** with degree >= N) < N
- Works because:
  – all other neighbors can be pushed on the stack before this node,
  – and then its degree is < N, so then it can be pushed
  – <u>Example</u>: (N = 2)

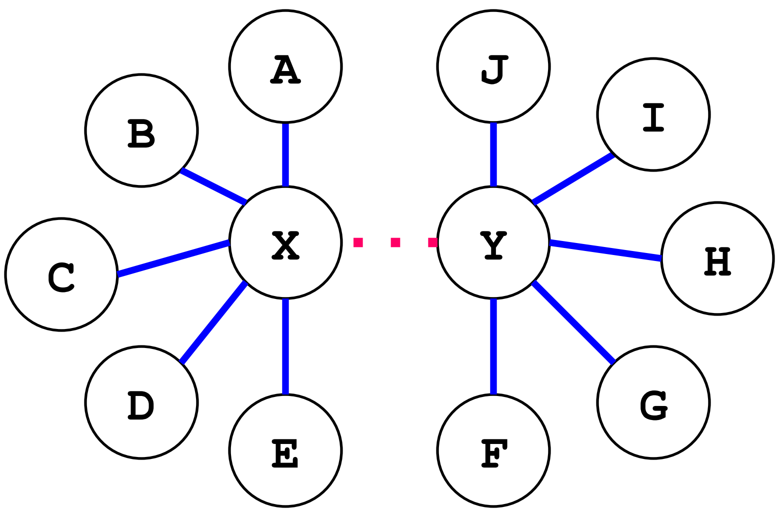






Briggs's Algorithm

- Nodes **X** and **Y** can be coalesced if:
  - (number of neighbors of **X/Y** with
  - degree >= N) < N
- <u>More extreme example</u>: (N = 2)

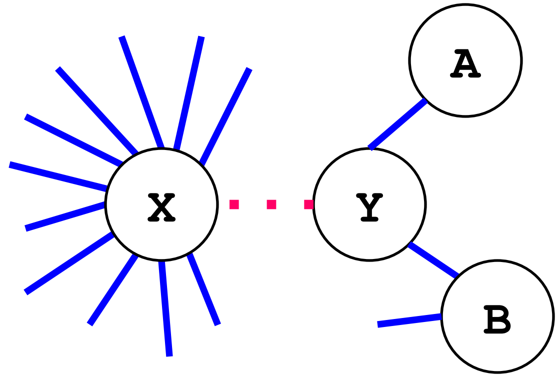


X/Y
J
I
H
G
F
E
D
C
B
A

George's Algorithm

<u>Motivation</u>:

- imagine that **`X`** has a very high degree, but **`Y`** has a much smaller degree
  - (perhaps because **`X`** has a large live range)

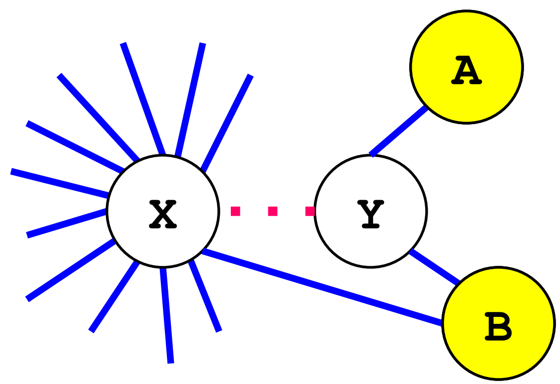


- With Briggs's algorithm, we would inspect all neighbors both **`X`** and **`Y`**
  - but **`X`** has a lot of neighbors!
- Can we get away with just inspecting the neighbors of **`Y`**?
  - showing that coalescing makes coloring no worse than it was given **`X`**?

George's Algorithm

- Coalescing **X** and **Y** does no harm if:
  - foreach neighbor **T** of **Y**, either:
    1. degree of **T** is <N, or ← *similar to Briggs: **T** will be pushed before **X/Y***
    2. **T** interferes with **X** ← *hence no change compared with coloring **X***

- Example: (N=2)



Summary

- *Coalescing* can enable register allocation to eliminate copy instructions
  - if both source and target of copy can be allocated to the same register
- However, coalescing must be applied with care to avoid causing register spilling
- Augment the interference graph:
  - dotted lines for coalescing candidate edges
  - try to allocate to same register, unless this may cause spilling
- Coalescing Algorithms:
  - simply based upon degree of coalescing candidate nodes (**`X`** and **`Y`**)
  - Briggs's algorithm
    - look at degree of neighboring nodes of **`X`** and **`Y`**
  - George's algorithm
    - asymmetrical: look at neighbors of **`Y`** (degree and interference with **`X`**)

CSC D70: Compiler Optimization Register Allocation & Coalescing

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The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons